Introducing Mechanistic Interpretability:

Demistify black boxes with **Circuit Analaysis**[1] & **Monosemanticity**[2]

J. Setpal

February {1, 8}, 2024

MACHINE LEARNING @ PURDUE

[1] https://transformer-circuits.pub/2021/framework/

[2] https://transformer-circuits.pub/2023/monosemantic-features/

Outline

1. Background & Intuition
2. Transformer Circuit Analysis
3. Towards Monosemanticity

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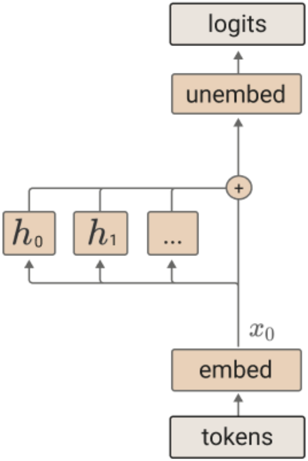
Today, we will interpret deep neural networks (transformer).

What will we Achieve Today?



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For mathematical simplicity, this model ignores biases, layer-norm and dense layers.

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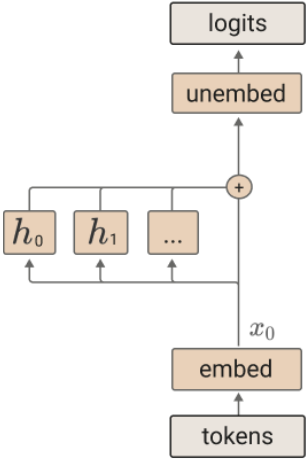

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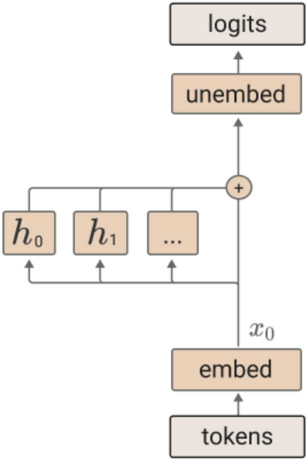

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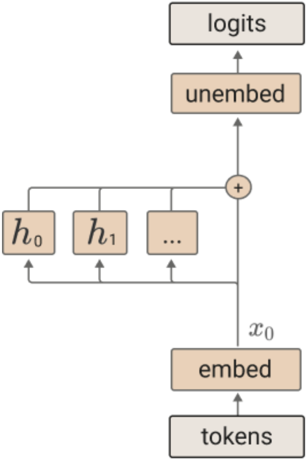

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- understand why attention works.
- observe recurring patterns in complex models.

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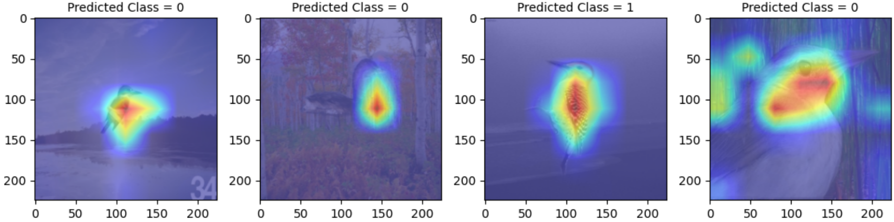
Most of interpretability seeks to extract representations from weights:



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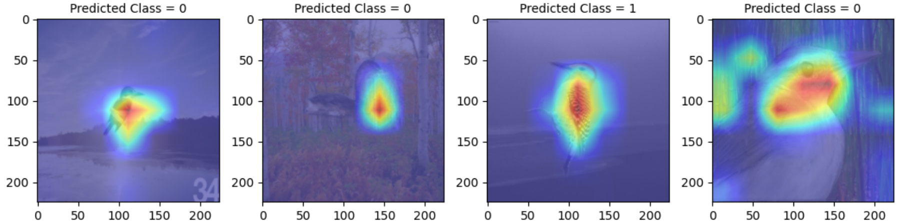

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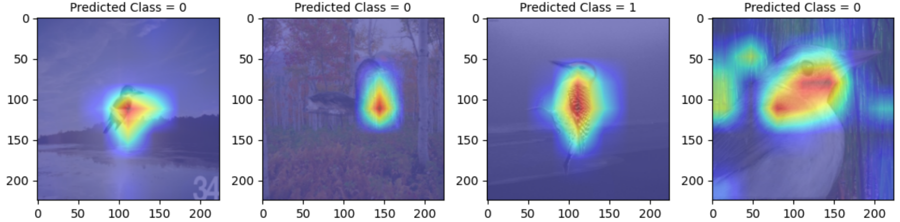

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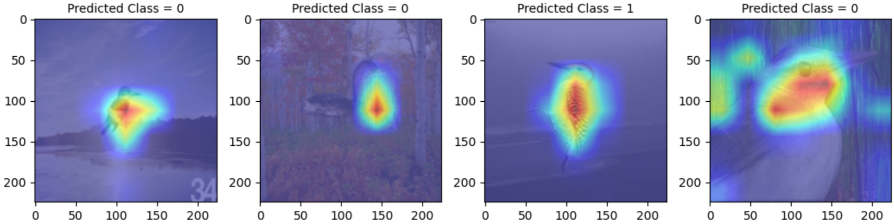

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It seeks to understand functions that *individual neurons* play in the inference of a neural network.

This can subsequently be used to offer high-level explanations for decisions, as well as guarantees during inference.

Outline

Self-Attention Synopsis

n-gram models used the following incorrect assumption:

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We query it to subset the important tokens. For $\{x_i\}_{i=1}^{t}$,

$$\alpha_i = \sigma_{softmax}\left(\frac{q_i k_i^T}{\sqrt{d_k}}\right) \quad (2)$$

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Reframing using Tensorization (1/3)

We can represent attention using **tensor products**:

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Observation: The equation is linear, <u>if we fix attention patterns</u>.

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And then apply them to unnormalized[3] attention:

$$A = \sigma_{softmax}\left([q_i k_j^T]_{i,j}\right) \tag{13}$$

$$= \sigma_{softmax}\left(t_0^T \cdot (I \otimes W_E^T W_Q^T) \cdot (I \otimes W_K W_E) \cdot t_0\right) \tag{14}$$

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Unravelling QK, OV Circuits (1/3)

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However, we're still missing one.

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b. The **Query-Key(QK) Circuit** $W_E^T W_Q^T W_K W_E$: determines which tokens to attend to.

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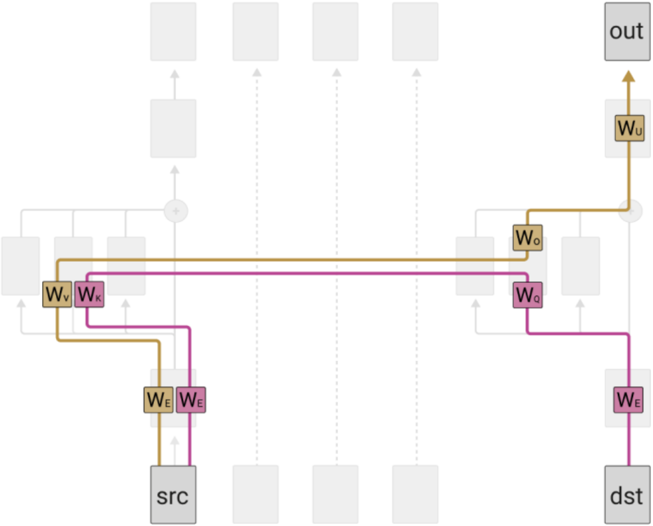

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From there, we look at the largest QK and OV entries.

Some examples of large entries QK/OV circuit

Source Token	Destination Token	Out Token	Example Skip Tri-grams
" perfect"	" are", " looks", " is", " provides"	" perfect", " super", " absolute", " pure"	" perfect... are perfect", " perfect... looks super"
" large"	" contains", " using", " specify", " contain"	" large", " small", " very", " huge"	" large... using large", " large... contains small"
" two"	" One", "\n ", " has", "\r\n ", "One"	" two", " three", " four", " five", " one"	" two... One two", " two... has three"
"lambda"	" $\\", "}{\\", "+\\", "(\\", " ${\\"	"lambda", "sorted", " lambda", "operator"	"lambda... $\\lambda", "lambda... +\\lambda"
"nbsp"	"&", " \"&", "}&", ">&", "=&"	"nbsp", "01", "gt", "00012", "nbs", "quot"	"nbsp... ", "nbsp... > "
"Great"	"The", " The", " the", " contains", " /"	" Great", " great", " poor", " Every"	"Great... The Great", "Great... the great"

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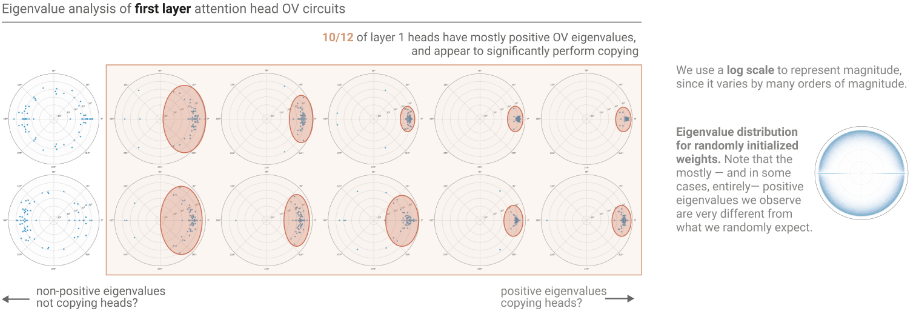

Importantly, note that positive eigenvalues mean they are copying 'on average', and are not definitive.

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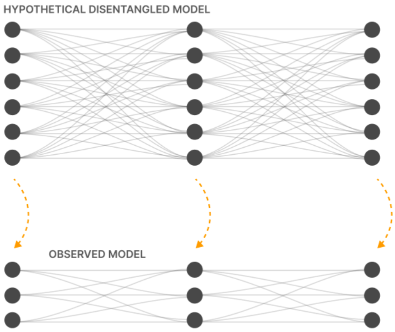

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Problem Setup

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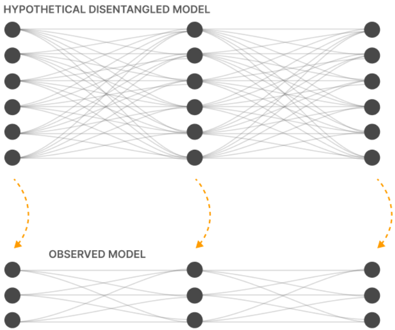

This is **superposition**.

When we perform an indvidual analysis of neurons, it fires for unrelated concepts.

This is **polysemanticity**.

Updated Architecture

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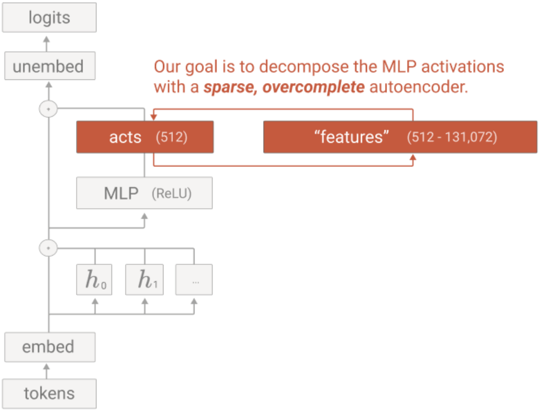
Let's instead analyze the following architecture *empirically*:



Training Setup

	Transformer	**Sparse Autoencoder**
Layers	1 Attention Block 1 MLP Block	1 ReLU 1 Linear
MLP Size	512	$512 \times f \in \{1, \ldots, 256\}^5$
Dataset	The Pile (100B tokens)	Activations (8B samples)
Loss	Autoregressive Log-Likelihood	$L2$ Reconstruction $L1$ on hidden-layer activation

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Objective: *polysemantic activations* $\xrightarrow{Tr}$ **monosemantic features**.

The sparse, overcomplete autoencoder is trained against this objective.

1. **Sparse** because we constrain activations (L1 penalty).
2. **Overcomplete** because the hidden layer exceeds the input dimension.

[5] $f = 8$ for our analysis

Sparse Dictionary Learning

Given $X := \{x^j\}_{j=1}^K; x_i \in \mathbb{R}^d$, we wish to find $D \in \mathbb{R}^{d \times n}, R \in \mathbb{R}^n$ s.t:

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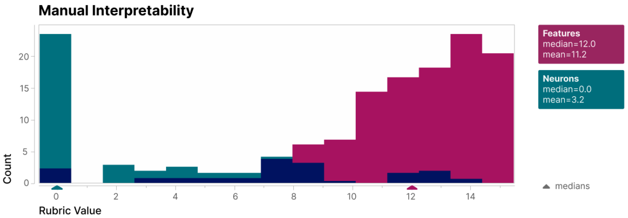
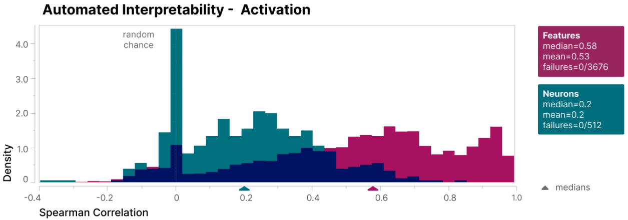
a. Training data $\propto n$(interpretable features).
b. Tying b_D before the encoder and after the decoder improves performance.
c. Dead neurons are periodically *resampled* to improve feature representations.

Evaluating Interpretability

Reliable evaluations on interpretability were scored based on a rubric:





Features were found to be interpretable when score > 8.

Analyzing Arabic Features

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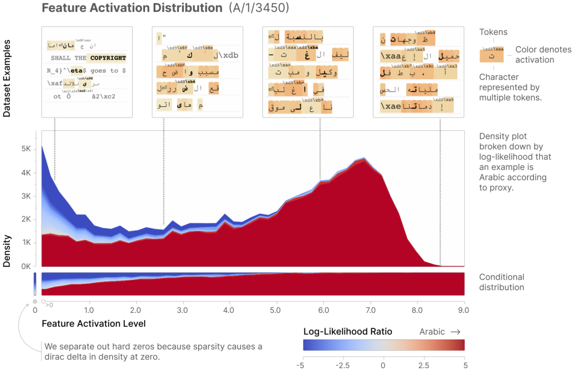
We can evaluate each token using the log-likelihood ratio:

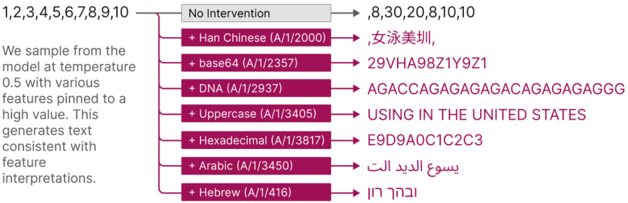
$$LL(t) = \log\left(P(t|\text{Arabic})/P(t)\right) \tag{20}$$

Despite representing 0.13% of training data, arabic script makes up **81% of active tokens**:



Pinned Feature Sampling

They can be used to steer generation.



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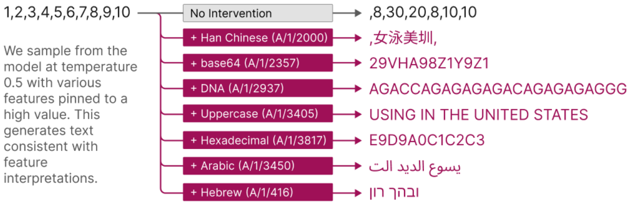

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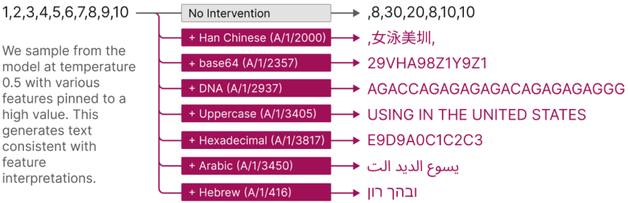

Approach: Set high values of features demonstrating desired behaviors, and then sample from the model.

We observe that interpreted features are actively used by the model.

Finite State Automaton

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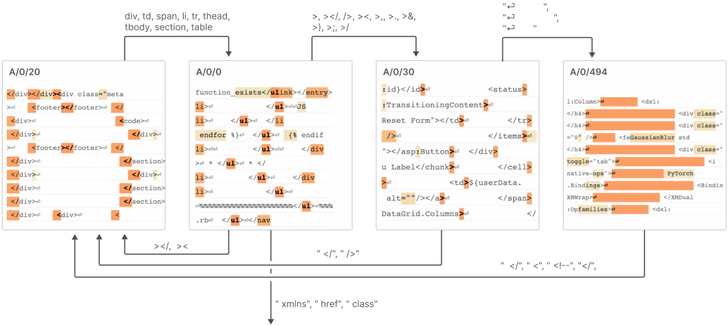
These present partial explanations of **memorizations** within transformers:



Reimplementation

If you can view this screen, I am making a mistake.

Thank you!

Have an awesome rest of your day!

Slides: `https://cs.purdue.edu/homes/jsetpal/slides/mechinterp.pdf`