DeepMind's AlphaTensor:

Discovering Faster Matrix Multiplication Algorithms with Reinforcement Learning

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Outline

1. Task Background
2. Algorithms as Tensor Decomposition
3. Neural Architecture Design

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What is Matrix Multiplication?

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 10 \end{bmatrix} \tag{1}$$

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Problem: Suboptimal!

What are Tensors?

Matrices, but *generalized*.

So, let's automate the Search Process!

There's a couple of considerations:

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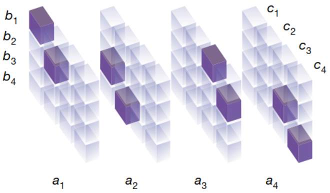
Let's Restructure the Problem!

a

$$\begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \cdot \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$$



b

$$m_1 = (a_1 + a_4)(b_1 + b_4)$$
$$m_2 = (a_3 + a_4)\, b_1$$
$$m_3 = a_1\, (b_2 - b_4)$$
$$m_4 = a_4\, (b_3 - b_1)$$
$$m_5 = (a_1 + a_2)\, b_4$$
$$m_6 = (a_3 - a_1)(b_1 + b_2)$$
$$m_7 = (a_2 - a_4)(b_3 + b_4)$$
$$c_1 = m_1 + m_4 - m_5 + m_7$$
$$c_2 = m_3 + m_5$$
$$c_3 = m_2 + m_4$$
$$c_4 = m_1 - m_2 + m_3 + m_6$$

c

$$\mathbf{U} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{W} = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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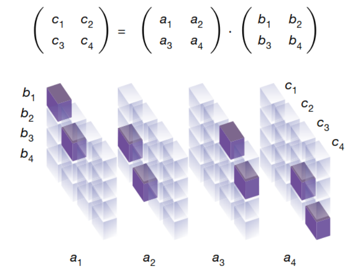

b

$$
\begin{aligned}
m_1 &= (a_1 + a_4)(b_1 + b_4) \\
m_2 &= (a_3 + a_4)\, b_1 \\
m_3 &= a_1\, (b_2 - b_4) \\
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c_1 &= m_1 + m_4 - m_5 + m_7 \\
c_2 &= m_3 + m_5 \\
c_3 &= m_2 + m_4 \\
c_4 &= m_1 - m_2 + m_3 + m_6
\end{aligned}
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\mathbf{U} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}
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$$
\mathbf{V} = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 \end{pmatrix}
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$$
\mathbf{W} = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}
$$

This can then be represented using:

$$
\mathcal{T}_n = \sum_{r=1}^{R} u^{(r)} \otimes v^{(r)} \otimes w^{(r)}
$$

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Attention is All It Needs!

Leveraging the Current tensor and the history, embedding, policy and value head.

Synthetic Demonstration

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Randomly selected tensor pairs are used to compose a 3D-tensor. This generates synthetic data on which the AlphaTensor agent trains.

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Change the basis is similar to a linear operation.

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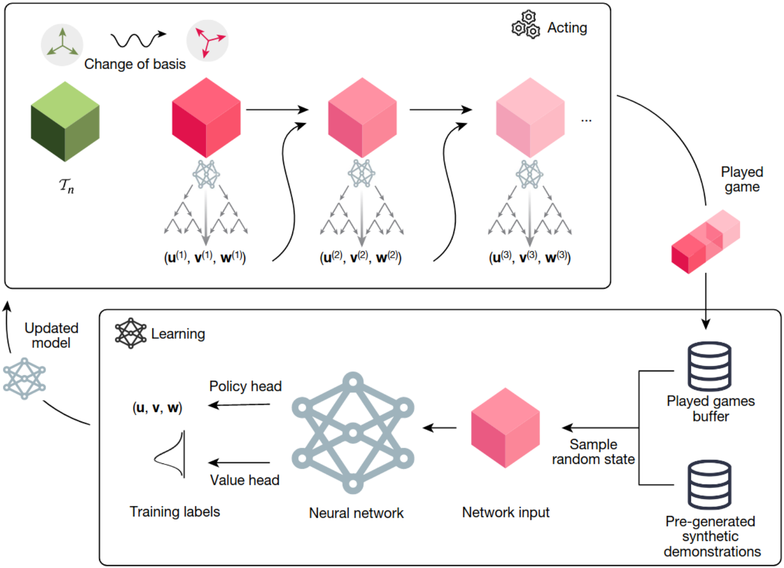
Easy way to augment existing data!

Putting it Together



Thank you!

Have an awesome rest of your day!

Slides: `https://cs.purdue.edu/homes/jsetpal/alphatensor.pdf`